

# PARAMETRIC CORRELATIONS BETWEEN EXPERIMENTAL RESULTS AND THE BASE ISOLATION, IN SITU, STRUCTURAL ONES

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**Abstract.** The paper addresses the topic of the inconsistency between experimental, laboratory results for antiseismic devices and the dynamic stiffness, internal damping and dissipation parameters through additional devices. The necessary corrections of the stiffness and dissipation (damping) parameters will be presented when the antiseismic devices are equipped to satisfy the adequate functions in a complex structural system (buildings, viaducts, bridges) under the conditions of seismic motions characteristic to the Romanian territory. In this context, the kinematic excitation method, compared to the dynamic evaluation method of the vibration dissipation capacity, produced by a seismic shock is shown.

**Key words:** dynamic and kinematic excitation, laboratory testing, isolators

## **1. Introduction**

In accordance with the European Standard EN 15129:2009, the conformity assessment and CE marking of the anti-seismic elastomeric isolators, used as components of the base isolation systems, are done. Experimental research carried

out on specialized testing facilities, under dynamic regime, is conducted. The testing should reproduce the loading conditions equivalent to the operation specific parameters mainly defined by the geometrical and mechanical characteristics determining the damping

and stiffness parameters (Rivice, 1999; Meinovitch, 1990; Bratu, 2011a).

In order to determine the damping capacity, the elastomeric isolators are subjected to shearing by means of kinematic harmonic excitations defined under the form  $x = A_0 \sin \omega_1 t$ , where  $A_0$  is the absolute displacement amplitude of the loaded plate edge, in respect to the fixed edge and  $\omega_1$  is the kinematic excitation pulsation (Bratu, 2000; Kelly and Konstantinidis, 2011). In this case, considering of the hysteresis loop for the instantaneous viscoelastic force  $Q = kx + c\dot{x}$  depending on the instantaneous deflection  $x$ , the loss factor of the internal energy  $\eta$ , representing the dissipation effect, as well as the equivalent critical damping fraction  $\zeta_{eq}$  are determined (Giuliani, 1993; Tyler 1991).

The loading conditions showed that the isolator has no attached concentrated mass, meaning that  $m \equiv 0$ , and the excitation is kinematic exclusively with the harmonic displacement externally applied (Bratu and Dragan, 1997; Bratu and Vasile, 2012).

In this context, the damping expressed by the system parameter  $\zeta_{eq}$ , defined by

$$\zeta_{eq} = \frac{\Delta W}{4\pi W_{el}^{max}} \quad (\text{Bratu and Mihalcea, 2011}),$$

differs as compared with the parameter  $\zeta$  related to a linear viscoelastic system having the mass  $m \neq 0$ , expressed under

$$\zeta = \frac{c}{2\sqrt{km}} \quad (\text{Viola, 2001}).$$

Thus, the damping parameter  $\zeta_{eq}$  could be determined by laboratory testing only. Thus, the actual structural analysis using the supporting and the base passive isolation system, having appropriate configuration, the values of parameter

$\zeta_{eq}$  in correlation with the effective parameter  $\zeta$  for the actual system, should be taken into account (Bratu and Vasile, 2010; Bratu, 2011c).

## 2. Exterior Harmonic Actions applied on the Elastomeric Isolator

This correlation between  $\zeta_{eq}$  obtained under harmonic elastic deflection actions kinematic applied and  $\zeta_d$  under dynamic actions could be performed, so that the energy dissipated under the kinematic regime  $\Delta W_c$  would be equal to the energy dissipated under dynamic regime  $\Delta W_d$ .

Figures 1 and 2 illustrate one elastomeric isolator consisting of several rubber layers separated by steel shims, under kinematic excitation of the form  $x(t) = A_0 \sin \omega_1 t$  (Fig. 1) and under dynamic excitation governed by the law  $F(t) = F_0 \sin \omega_2 t$  (Fig. 2). Fig. 3 and 4, represent a physical system with a symmetric excitation system. The linear viscoelastic system characteristics are  $c$  and  $k$ , without an added mass.

Under the kinematic excitation, of the form  $x(t) = A_0 \sin \omega_1 t$ , the dynamic response related to the viscoelastic connection force  $Q(t)$  is obtained as (Dolce *et.al.*, 2010; Faccioli and Paolucci, 2005):

$Q(t) = kx + c\dot{x} \equiv Q_0 \sin(\omega_1 t - \varphi_1)$  and further:

$$Q_0 = kA_0 \sqrt{1 + \eta_1^2} \quad (1)$$

$$\text{tg } \varphi_1 = \frac{c\omega_1}{k} = \eta_1 \quad (2)$$

The dissipated energy  $\Delta W_c$  can be written as:

$$\Delta W_c = \pi \omega_1 A_0^2$$

If one introduces  $c\omega_1 = k\eta_1$ , the previous relation becomes:

$$\Delta W_c = \pi k \eta_1 A_0^2 \quad (3)$$

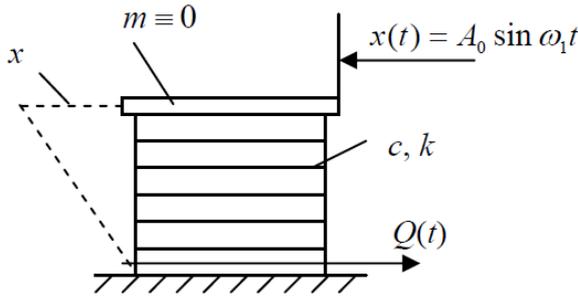


Fig. 1. Physical model for anti-seismic elastomeric isolator with asymmetrical excitation system: Kinematic excitation

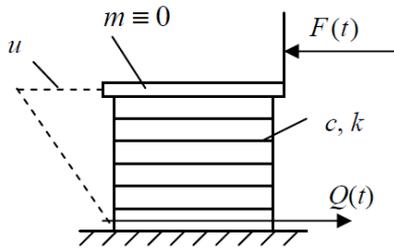


Fig. 2. Physical model for anti-seismic elastomeric isolator with asymmetrical excitation system: Dinamic excitation

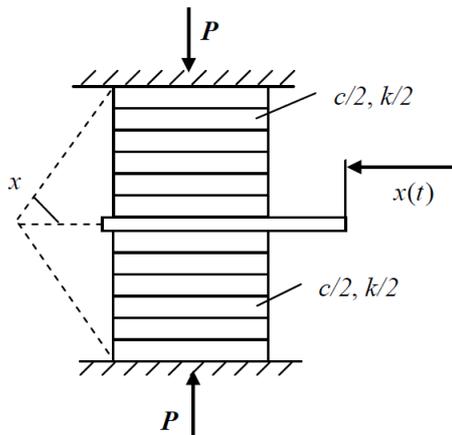


Fig. 3. Physical model with a symmetric excitation system: Kinematic excitation

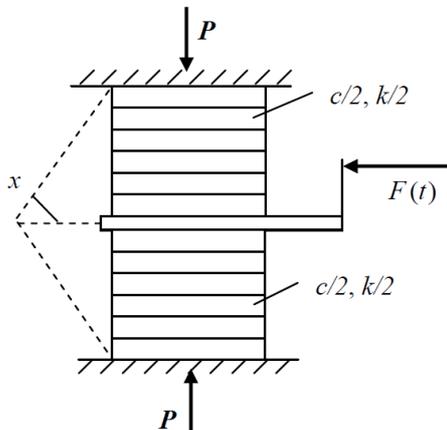


Fig. 4. Physical model with a symmetric excitation system: Dinamic excitation

### 3. Externally Applied Dynamic Force with $F_0$ Constant

Under the dynamic excitation of the form  $F(t) = F_0 \sin \omega_2 t$ , where  $F_0$  represents the amplitude of the action, the displacement response  $u = u(t)$  is obtained using the following instantaneous dynamic equilibrium equation:

$$c\dot{u} + ku = F_0 \sin \omega_2 t \quad (4)$$

Equation (4) underlines that one works with a 1<sup>st</sup> order physical system, without mass, so its resonance is less. The solution  $u(t) = A \sin(\omega_2 t - \varphi_2)$  should verify equation (4), leading to the following relation (Rivice, 2003):

$$A = \frac{F_0}{k} \frac{1}{\sqrt{1 + \eta_2^2}}, \text{tg } \varphi_2 = \frac{c\omega_2}{k} = \eta_2 \quad (5)$$

Taking into consideration relation (5), the dissipated energy  $\Delta W_d$  is given by the equation:

$$\Delta W_d = \pi k \eta_2 \frac{F_0^2}{k^2} \frac{1}{1 + \eta_2^2} \quad (6)$$

Using the condition that  $\Delta W_c = \Delta W_d$ , results in:

$$\eta_1 \eta_2^2 - \Psi^2 \eta_2 + \eta_1 = 0 \quad (7)$$

with  $\Psi = \frac{F_0}{kA_0}$  being the dynamic multiplication factor.

From (7) one obtains  $\eta_2$  under dynamic regime, depending on  $\eta_1$  under kinematic regime, as follows:

$$\eta_2 = \frac{1}{2\eta_1} \left[ \Psi^2 \pm \sqrt{\Psi^4 - 4\eta_1^2} \right] \quad (8)$$

a) the single solution of equation (8) is possible only under the following condition:

$$\Psi^4 - 4\eta_1^2 = 0 \quad (9)$$

and further

$$\Psi = \frac{F_0}{A_0 k} = \sqrt{2\eta_1} \quad (10)$$

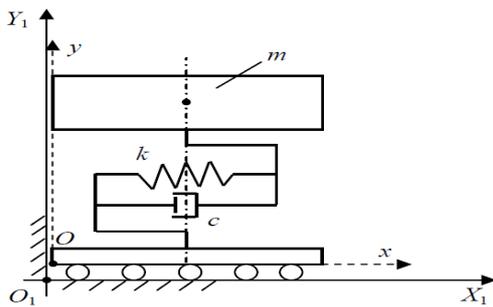
In this case, the solution of equation (8) is under the form:

$$\eta_2 = \frac{\Psi^2}{2\eta_1} = \frac{2\eta_1}{2\eta_1} = 1 \quad (11)$$

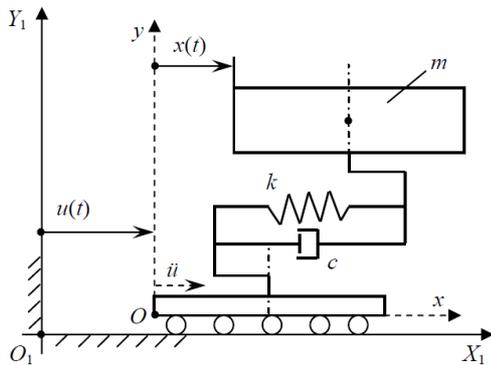
Thus, the loss factor has a unitary value and  $\zeta_2 = \frac{1}{2}\eta_2 = 0,5$ .

$\zeta_2^{\max} = 0,5$  represents the maximum value.

- b) the actual and distinct solutions of equation (8) are possible only for  $\Psi^4 - 4\eta_1^2 > 0$ , or  $\Psi > \sqrt{2\eta_1}$ . In case of actual parametric values  $0,2 \leq \eta_1 \leq 1,6$ , the range for  $\Psi$  is obtained as  $0,63 \leq \Psi \leq 1,78$ .



**Fig. 5.** Dynamic model of the passive isolation system under seismic inertia excitation: Instantaneous translation motion



**Fig. 6.** Dynamic model of the passive isolation system under seismic inertia excitation: Instantaneous acceleration

As an example, an elastomeric isolator without additional mass, with  $k = 1,5 \cdot 10^6 \text{ N/m}$ ,  $\zeta_{eq} = 0,20$ ,  $\eta_1 = 0,40$  is tested in laboratory under harmonic cycles having the linear amplitude  $A_0 = 0,08 \text{ m}$ . Under harmonic dynamic regime, characterized by  $F_0 = 120 \text{ kN}$ , the damping  $\eta_2$  is obtained as follows:

$$\Psi = \frac{F_0}{kA_0} = \frac{10^5}{1,5 \cdot 10^6 \cdot 8 \cdot 10^{-2}} = 1 > \sqrt{2 \cdot 0,4} = 0,89$$
 and from relation (8) it results in  $\eta_2'' = 2,0$

leading to  $\zeta_2' = 0,25$  and  $\zeta_2'' = 1,0$ , respectively, both values being higher than  $\zeta_{eq} = 0,20$ .

#### 4. Harmonic Seismic Actions on the Elastomeric Isolator

The dynamic model for the linear viscoelastic base isolation system, with  $m$ ,  $k$  and  $c$  considered as system parameters, is represented in Fig. 5 and 6, for two distinct positions. In Fig. 5, the system is under instantaneous translation motion. In Fig. 6, the coordinates  $x(t)$  related to the mass  $m$  and  $u(t)$  for the supporting base, caused by the seismic action with the instantaneous acceleration  $\ddot{u} = a_0 \sin \omega t$  for the fundamental spectral component, having the pulsation  $\omega$ , are illustrated (Carotti and Latella, 1999; Bratu, 2008).

The equation of motion for the mass  $m$ , related to the fixed reference system  $O_1X_1Y_1$  is of the form:

$$m\ddot{x}_1 + c\dot{x}_1 + kx_1 = 0 \quad (12)$$

in which  $x_1 = u + x$  represents the absolute displacement.

The relative displacement of the mass  $m$ , with respect to the moving reference system (having the acceleration  $\ddot{u}$ )  $Oxy$  is  $x(t)$ . Thus, one has:

$$m(\ddot{u} + \ddot{x}) + c\dot{x} + kx = 0$$

$$\text{or } m\ddot{x} + c\dot{x} + kx = -m\ddot{u} \quad (13)$$

If one introduces  $\ddot{u} = a_0 \sin \omega t$ , the previous relation becomes:

$$m\ddot{x} + c\dot{x} + kx = -ma_0 \sin \omega t \quad (14)$$

The final solution is:

$$x = A \sin(\omega t - \varphi) \quad (15)$$

in which  $A$  and  $\varphi$  are obtained from the condition that verifies equation (14).

Thus, it results:

$$A = \frac{a_0}{\omega_n^2 \sqrt{(\Omega^2 - 1)^2 + 4\zeta^2\Omega^2}} \quad (16)$$

$$\text{tg } \varphi = \frac{2\zeta\Omega}{1 - \Omega^2} \quad (17)$$

$$\Omega = \frac{\omega}{\omega_n}; \omega_n^2 = \frac{k}{m} \quad (18)$$

The energy dissipated under seismic dynamic excitation regime, is (Bratu *et. al*, 2011):

$$\Delta W_d = 2\pi\zeta \frac{m}{\omega_n^2} a_0^2 \frac{\Omega}{(\Omega^2 - 1)^2 + 4\zeta^2 \Omega^2} \quad (19)$$

The condition  $\Delta W_c = \Delta W_d$ , leads to the following relation:

$$\zeta_{eq} = \frac{\alpha^2 \zeta \Omega}{(\Omega^2 - 1)^2 + 4\zeta^2 \Omega^2} \quad (20)$$

and further we have the second order equation in  $\zeta$ , of the form:

$$4\Omega^2 \zeta_{eq} \zeta^2 + (\Omega^2 - 1)^2 \zeta_{eq} = \alpha^2 \Omega \zeta \quad (21)$$

having the solution:

$$\zeta = \frac{1}{8\Omega^2 \zeta_{eq}} \left[ \alpha^2 \Omega \pm \sqrt{\alpha^4 \Omega^2 - 16\Omega^2 (\Omega^2 - 1)^2 \zeta_{eq}^2} \right]$$

At resonance, for  $\Omega = 1$ , one obtains:

$$\zeta_{rez} = \frac{1}{4} \frac{\alpha^2}{\zeta_{eq}} \quad (22)$$

where  $\alpha = \frac{a_0}{A_0 \omega_n^2}$  is the acceleration

multiplication factor.

As an example, an elastomeric isolator having  $k = 1,5 \cdot 10^6$  N/m,  $\zeta_{eq} = 0,2$  is tested at  $A_0 = 0,088$ m. For a structural system with  $\omega_n = 2\pi$  subjected to a maximum acceleration  $a_0 = 0,25g$ , the result is:

$$\alpha = \frac{0,25 \cdot 10}{0,088 \cdot 4\pi^2} = 0,707$$

$$\zeta = \frac{1}{4} \frac{0,707^2}{0,2} = 0,62$$

meaning that  $\zeta$  for the system is three times higher than  $\zeta_{ech}$  determined in the laboratory.

## 6. Conclusions

The following conclusions can be synthesized based on the analysis intended to equalize the dissipated

energy under different excitation regimes, kinematic and dynamic:

- the testing method with kinematic excitation only, with harmonic displacement of the form  $x(t) = A_0 \sin \omega t$  allows the determination of the equivalent damping  $\zeta_{eq}$ . This is specific to the isolation system under laboratory experimental configuration, only.
- the damping evaluation under dynamic excitation for systems having actual dynamic behavior puts into evidence the necessity to determine the system parameter  $\zeta$  as a function of the experimental parameter  $\zeta_{eq}$ ;
- $\zeta$  for the system under actual configuration depends both on the type of the dynamic excitation as well as the existence of the mass with inertial effect.

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